CHAP 3 DIFF

1–46 □ Calculate y'.

1. $y = (x + 2)^8(x + 3)^6$
2. $y = \sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}}$
3. $y = \dfrac{x}{\sqrt{9 - 4x}}$
4. $y = \dfrac{e^x}{1 + x^2}$
5. $y = \sin(\cos x)$
6. $y = \sin^{-1}(e^x)$
7. $y = xe^{-1/x}$
8. $y = x^r e^{sx}$
9. $y = \tan\sqrt{1 - x}$
10. $y = \dfrac{1}{\sin(x - \sin x)}$
11. $y = \dfrac{x}{8 - 3x}$
12. $y = \left(x + \dfrac{1}{x^2}\right)^{\sqrt{7}}$
13. $y = \sec 2\theta$
14. $y = -2/\sqrt[4]{t^3}$
15. $y = (1 - x^{-1})^{-1}$
16. $y = \ln(\csc 5x)$
17. $y = e^{cx}(c \sin x - \cos x)$
18. $y = \ln(x^2 e^x)$
19. $y = e^{e^x}$
20. $y = 5^{x \tan x}$
21. $x^2y^3 + 3y^2 = x - 4y$
22. $x \tan y = y - 1$
23. $y = \sqrt[5]{x \tan x}$
24. $y = \sec(1 + x^2)$
25. $x^2 = y(y + 1)$
26. $y = 1/\sqrt[3]{x + \sqrt{x}}$
27. $y = \dfrac{(x - 1)(x - 4)}{(x - 2)(x - 3)}$
28. $y = \sqrt{\sin\sqrt{x}}$
29. $y = \log_{10}(x^2 - x)$
30. $y = e^{\cos x} + \cos(e^x)$
31. $y = \ln \sin x - \frac{1}{2}\sin^2 x$
32. $y = \arctan(\arcsin\sqrt{x})$
33. $y = \sin(\tan\sqrt{1 + x^3})$
34. $xe^y = y - 1$
35. $y = \cot(3x^2 + 5)$
36. $y = \dfrac{(x + \lambda)^4}{x^4 + \lambda^4}$
37. $y = \cos^2(\tan x)$
38. $y = \dfrac{\sin mx}{x}$
39. $y = \dfrac{\sqrt{x + 1}\,(2 - x)^5}{(x + 3)^7}$
40. $y = \ln|\csc 3x + \cot 3x|$
41. $y = x \sinh(x^2)$
42. $y = x^{\cos x}$
43. $y = \ln(\cosh 3x)$
44. $y = \ln\left|\dfrac{x^2 - 4}{2x + 5}\right|$
45. $y = \cosh^{-1}(\sinh x)$
46. $y = x \tanh^{-1}\sqrt{x}$

47. If $f(x) = 1/(2x - 1)^5$, find $f''(0)$.
48. If $g(t) = \csc 2t$, find $g'''(-\pi/8)$.
49. Find y'' if $x^6 + y^6 = 1$.
50. Find $f^{(n)}(x)$ if $f(x) = 1/(2 - x)$.
51. Use mathematical induction to show that if $f(x) = xe^x$, then $f^{(n)}(x) = (x + n)e^x$.
52. Evaluate $\lim_{t \to 0} \dfrac{t^3}{\tan^3 2t}$.

53–57 □ Find an equation of the tangent to the curve at the given point.

53. $y = \dfrac{x}{x^2 - 2}$, $(2, 1)$
54. $\sqrt{x} + \sqrt{y} = 3$, $(4, 1)$
55. $y = \tan x$, $(\pi/3, \sqrt{3})$
56. $y = x\sqrt{1 + x^2}$, $(1, \sqrt{2})$
57. $y = \ln(e^x + e^{2x})$, $(0, \ln 2)$

58. If $f(x) = xe^{\sin x}$, find $f'(x)$. Graph f and f' on the same screen and comment.
59. (a) If $f(x) = x\sqrt{5 - x}$, find $f'(x)$.
    (b) Find equations of the tangent lines to the curve $y = x\sqrt{5 - x}$ at the points $(1, 2)$ and $(4, 4)$.
    (c) Illustrate part (b) by graphing the curve and tangent lines.
    (d) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.
60. (a) If $f(x) = 4x - \tan x$, $-\pi/2 < x < \pi/2$, find f' and f''.
    (b) Check to see that your answers to part (a) are reasonable by comparing the graphs of f, f', and f''.
61. At what points on the curve $y = \sin x + \cos x$, $0 \leq x \leq 2\pi$, is the tangent line horizontal?
62. Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope 1.
63. If $f(x) = (x - a)(x - b)(x - c)$, show that
$$\frac{f'(x)}{f(x)} = \frac{1}{x - a} + \frac{1}{x - b} + \frac{1}{x - c}$$
64. (a) By differentiating the double-angle formula
$$\cos 2x = \cos^2 x - \sin^2 x$$
obtain the double-angle formula for the sine function.